ON THE DISTRIBUTION OF PISOT AND CNS POLYNOMIALS

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Abstract

This talk is based on two joint papers with S. Akiyama, H. Brunotte and J. Thuswaldner [1, 2]. Let M denote an integer and $P(X) = p_d x^d + p_{d-1}X^{d-1} + \cdots + p_0$ a polynomial with integer coefficients with $p_d = 1$. Let $\mathcal{B}_d(M)$ and $\mathcal{B}_d^0(M)$ denote the set of coefficient vectors $(p_{d-1}, \ldots, p_0)$ of Pisot or Salem polynomials as well as Pisot polynomials with property (F) such that $M \le p_{d-1} < 0$. It is clear that both sets are finite.

Assume now $p_0 \ge 2$ and set $\mathcal{N} = \{0, 1, \ldots, p_0 - 1\}$. Furthermore, denote the image of X under the canonical epimorphism from $\mathbb{Z}[X]$ to $R := \mathbb{Z}[X]/P(X)\mathbb{Z}[X]$ by x. Since $p_d = 1$ it is clear that each coset of R has a unique element of degree at most $d-1$, say

$$A(X) = A_{d-1}X^{d-1} + \cdots + A_1 X + A_0 \quad (A_0, \ldots, A_{d-1} \in \mathbb{Z}).$$

Let $\mathcal{G} := \{A(X) \in \mathbb{Z}[X] \ : \ \deg A < d\}$ and

$$T_P(A) = \sum_{i=0}^{d-1} (A_{i+1} - q p_{i+1}) X^i,$$

where $A_d = 0$ and $q = \lfloor A_0/p_0 \rfloor$. Then $T_P \ : \ \mathcal{G} \to \mathcal{G}$ and

$$A(x) = (A_0 - q p_0) + x T_P(A), \text{ where } A_0 - q p_0 \in \mathcal{N}.$$

If for each $A \in \mathcal{G}$ there is a $k \in \mathbb{N}$ such that $T_P^k(A) = 0$ we call P a *canonical number system polynomial* (*CNS polynomial* for short).

Associated to the notion of CNS we define for each $d \in \mathbb{N}$, $d \ge 1$ the sets

$$\mathbb{C}_d := \{(p_0, \ldots, p_{d-1}) \in \mathbb{Z}^d \ : \ |p_0| \ge 2 \text{ and } T_{X^d + p_{d-1}X^{d-1} + \cdots + p_0} \text{ has only finite orbits}\}$$

and

$$\mathbb{C}_d^0 := \{(p_0, \ldots, p_{d-1}) \in \mathbb{Z}^d : |p_0| \ge 2 \text{ and } \forall A \in \mathcal{G}\ \exists \ell \in \mathbb{N} : T^{\ell}_{X^d + p_{d-1}X^{d-1} + \cdots + p_0}(A) = 0\}.$$

For $M \in \mathbb{N}_{>0}$ we set

$$(1) \qquad \mathcal{C}_d(M) := \left\{ \left(\frac{p_{d-1}}{M}, \ldots, \frac{p_1}{M} \right) \in \mathbb{R}^{d-1} \ : \ (M, p_1, \ldots, p_{d-1}) \in \mathcal{C}_d \right\}$$

and

$$(2) \qquad \mathcal{C}_d^0(M) := \left\{ \left(\frac{p_{d-1}}{M}, \ldots, \frac{p_1}{M} \right) \in \mathbb{R}^{d-1} \ : \ (M, p_1, \ldots, p_{d-1}) \in \mathcal{C}_d^0 \right\}.$$

In the cited papers we proved asymptotic formulae for the size of the sets $\mathcal{B}_d(M), \mathcal{C}_d^0(M), \mathcal{D}_d(M)$ and $\mathcal{C}_d^0(M)$. The main term is always of the form cM^{d-1} with a constant c, which is the measure of appropriately defined set. In the talk we outline the basic steps of the proofs and suggest some open problem.

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References

[1] S. Akiyama, H. Brunotte, A. Pethő and J. M. Thuswaldner, *Generalized radix representations and dynamical systems III*, Osaka J. Math. **45** (2008), 347 – 374.

[2] S. Akiyama, H. Brunotte, A. Pethő and J. M. Thuswaldner, *Generalized radix representations and dynamical systems IV*, Indagationes Mathematicae, to appear.