On the average of some q-additive functions

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Let q be an arbitrary fixed natural number ≥ 2. A complex-valued arithmetical function $g(n)$ is called *q-additive*, if $g(n)$ satisfies the relations

$$g(0) = 0 \quad \text{and} \quad g(n) = \sum_{k=0}^{\infty} g(a_k(n)q^k), \quad n \in \mathbf{N},$$

whenever n has the q-adic expansion

$$n = \sum_{k=0}^{\infty} a_k(n)q^k, \quad 0 \leq a_k(n) \leq q-1.$$

We introduce an weight function $f : \mathbf{N} \cup \{0\} \to \mathbf{C}$ and define the arithmetic function $g_f(n)$ by

$$g_f(0) = 0 \quad \text{and} \quad g_f(n) = \sum_{k=0}^{\infty} a_k(n) f(k), \quad n \in \mathbf{N}. \tag{1}$$

Then $g_f(n)$ is a q-additive function associated with the weight function $f(k)$. If we choose $f(k) = 1$ for $k \in \mathbf{N}$, then $g_f(n)$ is *the function sum of digits*. The function sum of digits has been studied by some mathematicians.

In this talk we discuss m-tuple averages of $g_f(n)$ from a function theoretical point of view. We also consider an application based on m-tuple averages of $g_f(n)$.