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*S-unit equations in number fields: effective results, generalizations,
applications, abc conjecture*

The S -unit equations have many important applications and generalizations. In our talk we first present some explicit bounds, obtained jointly with K. Yu, for the solutions of S -unit equations in two unknowns in algebraic number fields. Recently these bounds were used by the speaker to derive the best known effective estimates in the direction of the uniform abc conjecture over number fields. We then formulate a recent effective finiteness theorem, due to Pintér and the speaker, for a common generalization of S -unit equations and binomial Thue-equations with unknown exponents. This provides a general effective result for the corresponding three-parameter family of S -unit equations. Finally, another generalization will be discussed for polynomial equations in two unknowns with algebraic coefficients. In joint work with Bérczes, Evertse and Pontreau we have recently derived explicit upper bounds for the solutions of such equations, when the unknowns are taken from algebraic numbers that, with respect to the height, are "close" to a given multiplicative subgroup of finite rank of $\overline{\mathbb{Q}}^*$. The main feature of these results is that the solutions under consideration do not have to lie in a prescribed number field. Hence we gave bounds not only for the heights but also for the degrees of the solutions. These results of ours generalize a theorem of Bombieri and Gubler. Further, for the special class of varieties we consider, they can be regarded as effective versions of some very general but ineffective theorems of Laurent, Poonen, Evertse, Schlickewei, Schmidt and Rémond, respectively.