

# A certain estimate appearing in the Atkinson-type formula of cusp L-functions

AOKI Mitsuhiro (Nagoya)

Jutila[1] gave a new proof of Atkinson's formula, which is an estimation for the mean square of Riemann's zeta-function on the critical line, using the method of Laplace transforms. He also demonstrated that cusp  $L$ -functions attached to  $SL_2(\mathbb{Z})$  and weight  $k$  have an analogue of Atkinson's formula :

$$E(T) = \int_0^T Z\left(\frac{1}{2} + it\right) dt$$

Remark 1: The analytic function  $Z(s), (s \in \mathbb{C})$  is a product of the cusp  $L$ -function and an entire function.

Remark 2 : In this case, we have no "main" term.

We are interested in the error term  $E(T)$  and already showed an estimation analogue of Heath-Brown[2]. Today we will show  $E(T)$  changes its sign as an analogue of a result of Heath-Brown and Tsang[3].

- [1] M. Jutila, "Atkinson's formula revisited", in *Voronoi's Impact on Modern Science, Book 1*, P. Engel and H. Syta (eds.), National Academy of Sciences of Ukraine, Inst. Math., Kyiv (1998), pp.137-154
- [2] D. R. Heath-Brown, "The mean value theorem for the Riemann zeta-function", *mathematica*, **25** (1978), 177-184
- [3] D. R. Heath-Brown and K. Tsang, "Sign Changes of  $E(T)$ ,  $\Delta(x)$ , and  $P(x)$ ", *J. Number Theory*, **49** (1994), 73-83